

ISBN : 978-93-91077-53-2

# Pre-Conference Proceedings of the International Conference on Discrete Mathematics (ICDM2021-MSU)

Editors : R. Kala, S. Monikandan, K. Selvakumar  
and T. Tamizh Chelvam



Department of Mathematics  
Manonmaniam Sundaranar University  
Tirunelveli 627012, Tamilnadu, INDIA



37	<i>C. Sankari, R. Sangeetha and K. Arthi</i> Claw – decomposition of Kneser graph $KG_{n,3}$	247 – 253
38	<i>P. Arul Paul Sudhahar and W. Jebi</i> Nonsplit edge monophonic domination number of a graph	254 – 260
39	<i>S.K. Vaidya and G.K. Rathod</i> On general randic energy of graphs	261 – 268
40	<i>Sheeba Agnes and R. Geethanjaliyadav</i> Rupture degree of some classes of graphs	269 – 279
41	<i>P. Arul Paul Sudhahar and J. Jeba Lisa</i> Minimal nonsplit geodetic domination number of a graph	280 – 287
42	<i>K. Selvakumar and P. Subbulakshmi</i> On the crosscap of a graph associated to a commutative ring	288 – 298
43	<i>S.L. Sumi, V. Mary Gleeta and J. Befija Minnie</i> The geodetic cototal domination number of a graph	299 – 307
44	<i>K. Prabha Ananthi and T. Tamizh Chelvam</i> On the reduced non - zero component graph of semi-modules	308 – 316
45	<i>S. Sujitha and D. Sharmila</i> The Adjacency Energy of a T2 Hypergraph	317 – 323
46	<i>S. Santha and G.T. Krishna Veni</i> Inverse domination of the transformation of n-sunlet graph	324 – 327
47	<i>S. Sujitha and L. Mary Jenitha</i> The 3 – component connectivity number of an arithmetic graph	328 – 333
48	<i>K. Reji Kumar and E.N. Satheesh</i> On the edge – vertex roman domination of graphs	334 – 339
49	<i>J. Ashwini, S. Pethanachi Selvam and R. B. Gnanajothi</i> Various versions of lucky labelling of book graphs	340 – 346
50	<i>C.M. Deshpande, Y.S. Gaidhani and B.P. Athawale</i> Circulant semigraphs	347 – 354
51	<i>Anjali Yadav and S. Minirani</i> Distance antimagic labelling of some graphs	355 – 357
52	<i>S.S. Sandhya and H. Aswathy</i> More results on root square mean labelling of graphs	358 – 359
53	<i>T. Sathiyandham and S. Arokiaraj</i> C – average eccentric graphs	360 – 365
54	<i>T. Tamizh Chelvam, M. Balamurugan and S. Anukumar Kathirvel</i> Connectivity of the complement of generalized total graph of finite fields	366 – 370
55	<i>B. Chandra and R. Kala</i> Group S3 cordial prime labeling of some Shadow graphs	371 – 383
56	<i>M. Sree Vidya and S.S. Sandhya</i> Double continuous monotonic decomposition of Stolarsky 3 mean graphs	384 – 389
57	<i>C. Gayathri and S. Saravanakumar</i> Equitable Irregular Edge-Weighting of Polar Grid Graph and Mongolian Tent Graph	390 – 396

## The Adjacency Energy of a $T_2$ Hypergraph

*S. Sujitha<sup>1</sup> and D. Sharmila<sup>2</sup>*

<sup>1</sup> Department of Mathematics

Manonmaniam Sundaranar University

Tirunelveli, Tamil nadu, India

<sup>2</sup> Department of Mathematics

Holy Cross College (Autonomous), Nagercoil, India

<sup>1</sup> sharmilareegan10@gmail.com, <sup>2</sup>sujitha.s@holycrossngl.edu.in

### Abstract

A hypergraph  $H = (X, D)$  is said to be a  $T_2$  hypergraph if for any three distinct vertices  $u, v$  and  $w$  in  $H$ , there exist a hyperedge containing  $u, v$  but not  $w$  and another hyperedge containing  $w$  but not  $u$  and  $v$ . In this article, the adjacency energy of a  $T_2$  hypergraph is studied. It is shown that,  $\epsilon(H) > \sqrt{\frac{w(H)}{n}}$ .

**Keywords:**  $T_2$  hypergraph, Adjacency matrix, Adjacency energy.

**Subject Classification:** 05C65

## 1 Introduction

The basic definitions and terminologies of a hypergraph are not given here and we refer it [3] and [13]. The concept of hypergraph was introduced by Berge in 1967. Later the same concept was studied by different authors in [13] and [1], Seena V and Raji Pilakkat were introduced Hausdroff hypergraph,  $T_0$  hypergraph and  $T_1$  hypergraph. Based on [10] and [11] we introduced a new class of hypergraph namely  $T_2$  hypergraph is studied the parameter adjacency energy of a hypergraph. Throughout this article  $H = T_2$  is a simple connected hypergraph with order  $n$  and size  $m$ . Here the order and size are the minimum number of vertices and edges used to define a  $T_2$  hypergraph. The following definitions and theorems are used in sequel.

**Definition 1.1.** [9] A hypergraph  $H = (X, D)$  is said to be a Hausdroff hypergraph if for any two distinct vertices  $u, v$  of  $X$  there exist hyperedges  $D_1$  and  $D_2 \in D$  such that  $u \in D_1, v \in D_2$  and  $D_1 \cap D_2 = \phi$ .

**Definition 1.2.** [10] A hypergraph  $H = (X, D)$  is said to be a  $T_0$  hypergraph if for any two distinct vertices  $u$  and  $v$  of  $X$  there exist a hyperedge containing one of them but not the other.

**Definition 1.3.** [11] A hypergraph  $H = (X, D)$  is said to be a  $T_1$  hypergraph if for any two distinct vertices  $u$  and  $v$  of  $X$  there exist a hyperedge containing  $u$  but not  $v$  and another hyperedge containing  $v$  but not  $u$ .

**Definition 1.4.** [2] The Wiener index  $W(H)$  is defined by  $W(H) = \sum_{u,v \in X(H)} d_H(u, v)$ .

**Definition 1.5.** [12] The Gutman index is defined as  $GutH = \sum_{u,v \in X(H)} d_H(u, v)d_H(u)d_H(v)$

**Definition 1.6.** [6] The rank of a hypergraph is the maximum cardinality of any of the edges in the hypergraph.

**Definition 1.7.** [6] The number of edges of a hypergraph  $H$  that are incident to a given vertex is called the degree of the vertex.

**Definition 1.8.** [7] The adjacency matrix is the square matrix which rows and columns are indexed by the vertices of  $H$  and where for all  $u, v \in X, u \neq v, a_{uv} = |\{d \in D/u, v \in D\}|$  and  $a_{uu} = 0$ .

**Theorem 1.9.** [4] For a graph  $G$  on  $n$  vertices and  $m$  edges,  $E(G) \leq \frac{2m}{n} + \sqrt{(n-1)[2m - (\frac{2m}{n})^2]}$ .

**Result 1.10.** (i) The minimum number of edges need to define a  $T_2$  hypergraph is  $m = \lceil \frac{2n+5}{4} \rceil$  where  $n$  is the number of vertices.

(ii) For a  $T_2$  hypergraph  $H$ , the minimum degree  $\delta(H) = 2$ .

(iii) For a  $T_2$  hypergraph  $H$ , rank  $r = \lceil \frac{2n+1}{4} \rceil$  where  $n \geq 3$ .

## 2 $T_2$ Hypergraph

Based on  $T_0$  and  $T_1$  hypergraphs, a class of hypergraph namely  $T_2$  hypergraph is defined in the following way and is observed some of its properties.

**Definition 2.1.** A hypergraph  $H = (X, D)$  is said to be a  $T_2$  hypergraph if for any three distinct vertices  $u, v$  and  $w$  in  $H$  there exist a hyperedge containing  $u$  and  $v$  but not  $w$  and another hyperedge containing  $w$  but not  $u$  and  $v$ .

**Result 2.2.** (i) The minimum number of edges need to define a  $T_2$  hypergraph is  $m = \lceil \frac{2n+5}{4} \rceil$  where  $n$  is the number of vertices.

(ii) For a  $T_2$  hypergraph  $H$ , the minimum degree  $\delta(H) = 2$ .

(iii) For a  $T_2$  hypergraph  $H$ , rank  $r = \lceil \frac{2n+1}{4} \rceil$  where  $n \geq 3$ .

## 3 The adjacency energy of a $T_2$ hypergraph

Adjacency energy of a hypergraph is introduced by Kaue Cardoso and Vilmar Trevisan in [6]. The definition is studied from [7]. In this section we find the Adjacency energy of a  $T_2$  hypergraph.

**Theorem 3.1.** If  $H = (X, D)$  is a  $T_2$  hypergraph with  $n$  vertices and  $m$  edges, then

$$\epsilon(H) > \sqrt{\delta \sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2}.$$

*Proof.*  $(\epsilon(H))^2 = \left(\sum_{i=1}^n \lambda_i\right)^2 \geq \sum_{i=1}^n \lambda_i^2 > 2 \sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2 > \delta \sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2$ .

$$\text{Thus } \epsilon(H) > \sqrt{\delta \sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2}. \quad \square$$

**Theorem 3.2.** Let  $H = (X, D)$  be a  $T_2$  hypergraph with maximum degree  $\Delta$ . If  $n \geq 5$  then  $\epsilon(H) \leq \sqrt{n(\Delta-1)G(H)} < n\sqrt{(\Delta^2-\Delta)W(H)}$  equality hold only if  $H$  is a  $T_2$  hypergraph with  $n=3$  and  $\Delta=2$ .

*Proof.* Let  $\lambda_1, \lambda_2, \dots, \lambda_n$  be the eigenvalues of  $A(H)$ , then

$$\begin{aligned} \sum_{i=1}^n \lambda_i^2 &= \sum_{i=1}^n \sum_{j=1}^n (a_{ij})^2 \leq \sum_{i=1}^n \left[ d(i)^2 + \sum_{j=1}^m (a_{ij})^2 \right] \\ &\leq (\Delta-1) \sum_{i=1}^n d(i)d(j)d(i,j) \leq (\Delta-1)G(H). \end{aligned}$$

$$\text{Now } \epsilon(H)^2 = \sum_{i=1}^n \lambda_i^2 \leq n \sum_{i=1}^n \lambda_i^2 \leq n(\Delta-1)G(H).$$

$$G(H) < n\Delta W(H)$$

$$G(H) \leq \sqrt{n(\Delta-1)G(H)} < \sqrt{n(\Delta-1)n\Delta W(H)}$$

$$G(H) \leq \sqrt{n(\Delta-1)G(H)} < n\sqrt{(\Delta^2-\Delta)W(H)}.$$

□

**Theorem 3.3.** *Let  $H$  be a  $T_2$  hypergraph with maximum degree  $\Delta$  and  $|v| = n$  then  $\epsilon(H) \leq \lambda_1 + \sqrt{(n-1)[(\Delta-1)G(H) - \lambda_1^2]}$ .*

*Proof.* We have  $\sum_{i=1}^n \lambda_i^2 = \lambda_1^2 + \sum_{i=2}^n \lambda_i^2$  and so  $\sum_{i=1}^n \lambda_i^2 \leq (\Delta-1)G(H)$

$$\epsilon(H) = \lambda_1 + \sum_{i=2}^n \lambda_i^2.$$

This gives that  $\epsilon(H) - \lambda_1 = \sum_{i=2}^n \lambda_i^2$ .

$$\begin{aligned} (\epsilon(H) - \lambda_1)^2 &\leq \left(\sum_{i=2}^n \lambda_i^2\right)^2 \\ &\leq (n-1) \sum_{i=2}^n \lambda_i^2 \\ &= (n-1)(\Delta-1)G(H) - \lambda_1^2 \\ \epsilon(H) - \lambda_1 &\leq \sqrt{(n-1)(\Delta-1)G(H) - \lambda_1^2} \\ \epsilon(H) &\leq \lambda_1 + \sqrt{(n-1)(\Delta-1)G(H) - \lambda_1^2}. \end{aligned}$$

□

**Theorem 3.4.** *If  $H$  is a  $T_2$  hypergraph with rank  $r$  and maximum degree  $\Delta$  then  $\epsilon(H) \geq \sqrt{n(\Delta-1)r}$  equality hold if  $H$  is a hypergraph with  $\Delta = m = n = 3$ .*

*Proof.* From lemma (25) in [6]

$$\begin{aligned} (\epsilon(H))^2 &\geq 2\left(\sum_{i=1}^n \lambda_i^2\right) \\ &\geq 2 \sum_{i=1}^n \sum_{j=1}^m (a_{ij})^2 \\ &\geq 2 \sum_{i=1}^n \sum_{j=1}^m (a_{ij}) \\ &\geq n(\Delta-1)r \\ \epsilon(H) &\geq \sqrt{n(\Delta-1)r}. \end{aligned}$$

□

**Theorem 3.5.** *If  $H$  is a  $T_2$  hypergraph then*

$$\epsilon(H) \geq \sqrt{\left(\sum_{i=1}^n \lambda_i^2\right) + \left(\frac{16m^2-48m+35}{4}\right)(\det A)^{\left|\frac{4}{4m-5}\right|}} \text{ equality hold if } m=3.$$

*Proof.* The result immediately follows from (lemma30) in [6].

□

**Theorem 3.6.** *Let  $H$  be a  $T_2$  hypergraph with rank  $r$  and  $|v| = n$  then  $\epsilon(H) \geq \sqrt{\frac{(r-1)^2}{n}W(H)}$  equality hold if  $H$  is a 3-uniform hypergraph with one edge.*

*Proof.* From (lemma25) in [6]

$$\begin{aligned}
 \epsilon(H)^2 &\geq 2 \sum_{i=1}^n \lambda_i^2 \\
 &\geq \sum_{i=1}^n \lambda_i^2 \\
 &= \sum_{i=1}^n \sum_{j=1}^m (a_{ij})^2 \\
 &\geq \sum_{i=1}^n \left| \frac{1}{n} \left( \sum_{i=1}^m (a_{ij})^2 \right) \right| \\
 &\geq 4 \frac{(r-1)^2}{n} W(H) \\
 &\geq 2 \sqrt{\frac{(r-1)^2}{n} W(H)}
 \end{aligned}$$

□

**Theorem 3.7.** Let  $H$  be a  $T_2$  hypergraph with rank  $r$  and  $|v| = n$  then  $\epsilon(H) \geq \sqrt{\frac{(r-1)^2}{n} W(H)}$  equality hold if  $H$  is a 3-uniform hypergraph with one edge.

*Proof.* From (lemma25) in [6]

$$\begin{aligned}
 \epsilon(H)^2 &\geq 2 \sum_{i=1}^n \lambda_i^2 \\
 &\geq \sum_{i=1}^n \lambda_i^2 \\
 &= \sum_{i=1}^n \sum_{j=1}^m (a_{ij})^2 \\
 &\geq \sum_{i=1}^n \left| \frac{1}{n} \left( \sum_{i=1}^m (a_{ij})^2 \right) \right| \\
 &\geq 4 \frac{(r-1)^2}{n} W(H) \\
 &\geq 2 \sqrt{\frac{(r-1)^2}{n} W(H)}
 \end{aligned}$$

□

**Theorem 3.8.** Let  $H$  be a  $T_2$  hypergraph with minimum degree  $\delta$  and  $|v|=n$  then  $\epsilon(H)^2 > \frac{W(H)}{n}$ .

*Proof.* From (lemma25) in [6]

$$\begin{aligned}
 \epsilon(H)^2 &\geq 2 \sum_{i=1}^n \lambda_i^2 \\
 &\geq \sum_{i=1}^n \lambda_i^2 \\
 &> \sum_{j=1}^n \left[ \frac{1}{m(m-1)} \left( \sum_{i=1}^n a_{ij} \right)^2 \right] \\
 &> \frac{W(H)}{n}.
 \end{aligned}$$

□

**Lemma 3.9.** *Let  $H$  be a  $T_2$  hypergraph with  $n \geq 4$  and  $m$  edges then  $\rho(A) \geq \frac{2n+13}{2(n-1)}$*

**Theorem 3.10.** *Let  $H$  be a  $T_2$  hypergraph with  $n \geq 4$  then  $\epsilon(H) \geq \frac{2n+13}{2(n-1)} + (n-1) + \ln |det A| - \ln \frac{2n+13}{2(n-1)}$  equality hold if  $H$  is a 3-uniform hypergraph with one edge.*

*Proof.* We have

$$\begin{aligned} \epsilon(H) &= \sum_{i=1}^n \lambda_i \\ &= \lambda_1 + \sum_{i=2}^n \lambda_i \\ &\geq \lambda_1 + (n-1) + \sum_{i=2}^n \ln |\lambda_i| \\ &= \lambda_1 + (n-1) + \ln \prod_{i=2}^n \lambda_i \\ &= \lambda_1 + (n-1) + \ln(det A) - \ln \lambda_1. \end{aligned}$$

From lemma 3.9

$$\epsilon(H) \geq \frac{2n+13}{2(n-1)} + (n-1) + \ln |det A| - \ln \frac{2n+13}{2(n-1)}.$$

□

**Theorem 3.11.** *Let  $H$  be a  $T_2$   $k$ -graph on  $n$  vertices and  $m$  edges. Then*

$$\epsilon(H) \leq \rho(A) + \sqrt{(n-1)(nm - \rho(A))},$$

where  $\rho(A)$  is the spectral radius of the adjacency matrix.

*Proof.* In  $T_2 - k$ -graph, by using Cauchy Schwart inequality

$$\begin{aligned} \sum_{i=1}^n \lambda_i &\leq nm \\ \rho(A) + \sum_{i=2}^n \lambda_i &\leq nm \\ \sum_{i=2}^n \lambda_i &\leq nm - \rho(A) \\ \epsilon(H) &\leq \rho(A) + \sum_{i=2}^n \lambda_i \\ &\leq \rho(A) + \sqrt{(n-1) \sum_{i=2}^n \lambda_i} \\ &\leq \sqrt{\rho(A) + (n-1)(nm - \rho(A))}. \end{aligned}$$

□

**Lemma 3.12.** *Let  $H$  be a connected  $T_2$   $k$ -graph and  $A(H)$  its adjacency matrix. The hypergraph  $H$  is regular iff  $x = (1 \ 1 \ 1 \dots \ 1)^T$*

**Theorem 3.13.** Let  $H$  be a connected  $T_2$   $k$ -graph and  $A(H)$  its adjacency matrix then  $\rho(A) \leq \frac{KW(H)}{\sqrt{n}}$  equality holds if  $H$  is regular.

*Proof.* In  $T_2$   $k$ -graph

$$\begin{aligned} \rho(A) &= \sqrt{\rho(A)^2} \\ &\leq \sqrt{X^T A^2 X} \\ &\leq \sqrt{\frac{(K \sum d(u, v))^2}{n}} \\ &= \frac{KW(H)}{\sqrt{n}}. \end{aligned}$$

□

**Remark 3.1.** Let  $H$  be a  $T_2$  hypergraph with rank  $r$ , minimum degree  $\delta$  and  $|v| = n$  then

- (i)  $\sum_{i=1}^n \lambda_i^2 \geq n(r-1)\delta(H)$
- (ii)  $\sum_{i=1}^n \lambda_i^2 \geq \frac{(r-1)^2}{3n} W(H)$ .
- (iii)  $\sum_{i=1}^n \lambda_i^2 \geq \frac{m}{m-1} \lambda_1$ .
- (iv)  $\sum_{i=1}^n \lambda_i^2 \geq \frac{nm}{n+m-1} \lambda_1$ .

**Theorem 3.14.** If  $H$  is a  $T_2$  hypergraph then

$$\epsilon(H) \geq \sqrt{(n(r-1))\delta(H) + \left(\frac{16m^2-48m+35}{4}\right)(\det A)^{\left|\frac{4}{4m-5}\right|}}.$$

*Proof.* Use theorem 3.5 and remark 3.1

□

## 4 Conclusion

In this article, the adjacency energy of a  $T_2$  hypergraph is obtained by using various graph parameters such as size  $m$ , rank  $r$ , Wiener index, Gutman index, maximum degree, minimum degree, spectral radius etc. In a similar way we can use other parameter also for calculating the adjacency energy of a  $T_2$  hypergraph.

## References

- [1] Alain Bretto, Hypergraph Theory: An Introduction, , Springer, Science & Bussiness Media, 2013.
- [2] A. A. Dobrynin, R. Entringer, I. Gutman Wiener index of trees:Theory and applications. Acta Appl. math. 66(2001), pp. 211-24.
- [3] Cluade Berge, Hypergraphs:Combinotorics of finite sets, vol. 4, Elsevier, 1984.
- [4] I. Gutman, (1978) The energy of a graph. Ber. Math, statist. Sekt. Forschungsz. Graz, 103, 1-22.
- [5] Kaue Cardoso and Vilmar Trevisan, Energies of hypergraphs, Electronic journal of Linear Algebra, ISBN 1081-3810, A publication of the International Linear Algebra, society volume 36, pp. 293-308 may 2020.
- [6] Kaue Cardoso and Vilmar Trevisan, Renata Del-vecchio, Lucas portugal, Adjacency Energies of hypergraphs, arxiv:2106. 07042v1[math. co]Jun2021.
- [7] K Reijkumar and Renny P Varghese, Spectrum of  $(k, r)$ -regular hypergraph'International Journal of Mathematical Combinatorics 2(2017), 52-59.



- [8] P. Nageshwari and P. B. Sarasija, A study on energy of graph , International Journal of Mathematical Analysis vol8, 2014, no 58, 2869-2871.
- [9] Seena V and Raji Pilakkat, Hausdorff hypergraphs, International of Applied Mathematics 29(2016), num1, 145-15.
- [10] Seena V and Raji Pilakkat,  $T_0$  hypergraphs, International of Applied Mathematics ISBN 0973-1768 volume13, num10(2017), PP. 7467-7478.
- [11] Seena V and Raji Pilakkat,  $T_1$  hypergraphs, International of Applied Mathematics ISBN 0973-1768 volume13, num10(2017), PP. 7453-7466.
- [12] Simon Mukwembi, On the upper bound of Gutman index of graphs, match commun math comput chem 68(2012)343-348 ISBN 0340-6253.
- [13] Vitaly Voloshin, Introduction to graph and hypergraph theory, Nova 2009.